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Solute Transport Modeling with Impact of Sinusoidal Form of Inlet Source at Boundary of the Geological Formation

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Abstract
The solute transport modeling is presented for the movement of various decay parameters under degradation situations of solute transport phenomena. In this present study analytical solution of solute transport modeling is presented for semi-infinite homogeneous geological formation using the Laplace transform technique. Uniform solute segmentation is assumed initially at the geological formation. The one end of the geological formation is polluted by temporally dependent sinusoidal source segmentation. At the other end of the domain solute segmentation flux is assumed to be zero. The concept of dispersion coefficient is directly proportional to the initial outflow velocity used for analytical results. The efforts of distinct velocity patterns (i.e.; exponential decreasing and sinusoidal) are significantly used to observe the solute segmentation behaviour. The solute segmentation distribution increases with time and decreases with space. The Relative Percent Error (RPE) is used to check the accuracy of the solute segmentation with respect to time period. The obtained results may be useful to maintain the quality of groundwater resources.

Keywords- Homogeneous, Solute, Contaminant, Porosities, Densities.

1. Introduction

Solute transport through geological formations from various direct and indirect source origins has been important research for the management of groundwater quality over the decades. Studying the solute contaminant transport problems through groundwater geological formations needs the use of suitable modeling tools. Abundant mathematical models have been developed to recognize the transport of contaminants through groundwater geological formations while considering the mass and transport of pollutants mixed in the groundwater reservoir. There is a partial differential equation called the Advection Dispersion Equation (ADE) which is extensively used in fields of solute segmentation modeling for effective demonstration for transport behavior of solute contaminants in groundwater systems. The solute spread in the direction of groundwater flow may acquire a higher segmentation than that of spreading against the direction of flux. Analytical solution of ADE is an important and effective mode with variability of usages, such as endorsed with numerical one, providing an estimated breakdown of solute segmentation, execution of sensitivity analysis for varied parameters strikes the advancement of contaminant segmentation. Batu (1989), Chen et al. (2008) and Gao et al. (2010) established the various closed-form solutions with the analysis of the effects of decay parameters for solute transport modeling equation for groundwater reservoirs. They have used seepage velocity is straight related to the preliminary outflow velocity and dispersion coefficient is straight related to the initial outflow velocity concept for obtained results. The various mathematical tools are used to solve solute transport modeling equations. Basha and El Habel (1993) discussed the exact result of a one-dimensional solute transport model equation with an analysed transient-dependent dispersion coefficient for an infinite groundwater geological formation. The investigation carried out is restricted to the asymptotic dispersion coefficient. It is evident from the literature that the higher mathematical models with time-dependent dispersion functions have not been explored so far to simulate solute transport through heterogeneous media. However, the key reactions for solute segmentation in groundwater reservoirs are symmetry-controlled sorption reactions and irreversible first-order reactions. Ahmadi et al. (1998) discussed the solute transport distribution model for describing the non-equilibrium dispersion phenomenon in heterogeneous groundwater geological formations. Also, a comparison of exact results with numerical experiments was demonstrated for stratified groundwater geological formation to describe most of the large-scale non-symmetry behaviour of bimodal heterogeneous structures, an extensive variety of Peclet numbers and diffusion/dispersion. This study was used to predict groundwater contamination in the different complex geometry of groundwater reservoirs.

The heterogeneity parameter in physical and chemical belongings controlled the segmentation of reactive solutes in groundwater geological formation. The spatial variation of physical and chemical heterogeneity is responsible for spatial-dependent solute transport. Srivastava et al. (2002) described exact solution of one-dimensional solute transport classical equation with spatial variation of reactive decay transport parameter. To represent the heterogeneity in different physical transport parameters they used an exponential increasing dispersivity function to illustrate various rates constants on the transport of groundwater.

However, for problems in groundwater remediation exact and approximate solutions are extended to depict the mass and segmentation of solute pollutants in groundwater systems. Singh and Das (2015) investigated the closed-form and numerical solutions of one-dimensional solute segmentation equations in semi-infinite heterogeneous geological formations by using a spatial dispersion coefficient. Moreover, closed-form and approximate solutions for exponential decreasing velocity distribution were compared with the root mean square method. This study was helpful in managing the quality of groundwater reservoirs.

However, Singh et al. (2016) established the exact and approximate solutions for inlet source contamination introduced at the splitting time domain for the finite length of the geological formation. The outcome result of solute contamination segmentation introduced for different velocity patterns. In general, solute contaminant segmentation is not easily established due to detection based on a relatively small volume of geological formation. Haslauer et al. (2017) described the nature of solute transport modeling over a dataset of different underlying processes, spatial dependent saturated hydraulic conductivity with macroscale and microscale for heterogeneous geological formation. The solute transport behaviour was influenced by the combined effects of each macroscale and microscale for the smallest time measure before the transverse mixing of solute occurs. Most of solute contaminants initiate from an unaccomplished arrangement of wastes into the ground surface in the form of point and non-point sources contamination. The reactive solute exists and their allied actions vary from zone to zone depending on the geological formation origins of the present bedrock. Chen et al. (2019) described the exact solution of the agro-eco-hydrological with solute transport equation subject to the equilibrium-controlled sorption. The developed closed-form solution measures the sorption mechanisms of each individual reactive transport equation, with lower segmentation predicted for an increased value of the kinetic sorption rate. This study illustrated that the reactive and non-reactive solutes controlled the solute contaminant segmentation into the layered soil formation.

However, Guleria and Swami (2018) investigated the approximate solution of mobile and immobile non-reactive solute transport equations with transient-dependent dispersion factors for the heterogeneous geological system. Moreover, the inherent uncertainty was estimated for solute segmentation in the mobile-immobile model with the use of the transient-dependent dispersion function. The transient dependent dispersion endorsed to the macro-level properties from the heterogeneity of sub-plane at the ground level. Das and Singh (2019) established closed-form exact and approximate results of solute transport equation with transient dispersion coefficient for distinct chemical constraints mounting into groundwater reservoir. They were found closed and numerical results of approximately the same nature. Moreover, Hosseini et al. (2020) developed the approximate result of a solute segmentation equation with variable density based on the framework of the prolonged finite element method to emulate the density-driven mass flow over the fractured geological formation. Moreover, the impacts of distinct constraints of the fracture geological formation, such as the pore and interconnectivity, such as the permeability and diffusion are also investigated. This study helps to signify the characteristics of the pore in the solute segmentation distribution.

Das et al. (2021) discussed a one-dimensional solute segmentation equation in the presence of source/sink term for a homogeneous semi-infinite saturated geological system. Moreover, the result of the solution was explored the influence of porosity, density and zero-order production terms in the groundwater geological formation. This study helps to the impact of distinct characteristics of groundwater reservoir to predict the solute segmentation along groundwater reservoirs. Nadella et al. (2023) expressed the approximate result of the one-dimensional unsteady solute segmentation equation for streams and channels enforced with several points loading with the impact of first-order decay term. The obtained results assist in catching the longitudinal variant of segmentation in a cell exactly as well as permit a good estimation of its longitudinal fluxes at the cell boundaries. Li et al. (2023) investigated the closed result for the solute transport model equation on the impact of diffusion, and adsorption in a large strain aquitard formation. They have significantly observed the larger leakage coefficient of aquitard through breakthrough time between the equivalent rigid medium and large strain medium. Many studies have been published concerning solute contaminant transport through geological formation. They are mixed to one another, mainly in terms of considered parameters and their dependency over space and time. Morel and Graf (2023) described the solute transport modeling behaviour in the presence of free advective density flow under the saturated condition of geological formation. They also analysed the solute transport nature for the unsaturated zone in respect of the infiltration rates potential of solute particles. Butler et al. (2023) introduced the impact of solute distribution patterns for both one-dimensional and radial flow in the presence of friction behaviour of dispersion. The impact of various dispersity was also explored to analyse the behaviour of solute patterns in heterogeneous geological formations for analytical and numerical results. Chang et al. (2024) explored the displacement variance of a two-dimensional solute transport problem for heterogeneous porous formation with the help of the stochastic method. They found the nature of solute transport in the mean flow direction increases with respect to hydraulic conductivity and thickness of the aquifer.

The solute exchange in the geological formation is a significant procedure influencing groundwater flow and solute contaminant flow. This paper aims to study the solute transport model with the impact of decay constraints for solid-liquid interphases inhomogeneous semi-infinite geological formations. Moreover, linear isotherm is described for inventing solute contaminant transport in between solid-liquid interphase. Initially, the geological formation was polluted with uniform initial source segmentation. At the inlet of the boundary, the transient sinusoidal source with a combination of uniform source segmentation is assumed whereas at the other boundary of the geological formation solute flux is taken to be zero. The analytical solution was developed by using Laplace Transform and by using distinct transformations. The solute segmentation pattern is developed for different temporally dependent velocity patterns with values of density and porosity values of distinct geological formations. Also, the solute segmentation is depicted to observe significantly the impact of various decay, and biodegradation parameters. The Relative Percent Error (RPE) is used to check the accuracy of the solute segmentation with respect to time period. The model is also validated with the data from existing research work. The obtained results are useful for controlling groundwater tools and management.

2. Mathematical Formulation

The solute contamination transport for geological formation is usually modelled by assuming a temporally dependent average outflow velocity and solute dispersion with impacts of various decay parameters in groundwater reservoirs. Mathematically, the differential equation for a semi-infinite homogeneous geological formation in solid-liquid interphase can be written as (Batu, 2005),

$$\frac{\partial c}{\partial t}+\frac{\rho}{\theta}\frac{\partial s}{\partial t}=D\frac{\partial^2 c}{\partial x^2}-\frac{u}{\theta}\frac{\partial c}{\partial x}-\lambda\frac{\rho}{\theta}s+\frac{q}{\theta}c \tag{1}$$

where, $D\left[L^2T^{-1}\right]$ is the spatially dispersion coefficient (i.e. demonstrating along the flow of groundwater), $c\left[ML^{-3}\right]$ is the dispersed solute segmentation in the liquid stage, $s\left[MM^{-1}\right]$ is the dispersed solute segmentation in the solid stage, $\rho\left[ML^{-3}\right]$ is the bulk solidity of the porous medium $u\left[LT^{-1}\right]$ is the unsteady sliding pore outflow velocity, $x\left[L\right]$ is a spatially direction of the groundwater flow, $t\left[T\right]$ is time, $\lambda\left[T^{-1}\right]$ is the first order decay constant, $q\left[T^{-1}\right]$ is the decay rate constant, θ is the porosity of the distinct geological formation such as slit, clay and gravel.

The model equation described the solute contaminant segmentation into the groundwater geological formation in the form of solid liquid interphase.

The simplest expression of linear isotherm for the relationships among the solid liquid interphase can be written as

$$s=k_d c \tag{2}$$

where, $k_d\left[L^3M^{-1}\right]$ is the distribution coefficient.

In this present study we considered that the initially the geological formation is not solute free.

At the starting of analysis i.e., at time $t = 0$ uniform solute segmentation initially is taken into consideration at time $t = 0$ into the geological formation.

$$c(x,t)=c_i; \quad x>0,\, t=0 \tag{3}$$

where, $c_i\left[ML^{-3}\right]$ initial solute segmentation.

As periodically movement of solute particles from subsurface to groundwater (e.g.; industrial waste material disposed on subsurface on periodic manure) some new solute particles may be successively added according to the strength of the source while some solute particle absorbed/adsorbed according to the strength of the solute. In order to analyse such type impact of sources one should consider sinusoidal source condition at the inlet of the boundary which monitors the changes in the groundwater quality and resources.

At the inlet of the geological formation i.e., at $x = 0$ the boundary of the domain affected by the temporally dependent sinusoidal source with the impact of uniform source segmentation is taken into consideration and mathematically expressed as:

$$c(x,t) = c_0(1-\sin\omega t); x = 0, t \geq 0 \tag{4}$$

where, $c_0\left[ML^{-3}\right]$ is uniform solute source segmentation and $\omega\left[T^{-1}\right]$ is biodegradation decay parameters.

At the other outlet of the geological formation i.e., $x \to \infty$ the solute segmentation flux is assumed to be zero

$$\frac{\partial c}{\partial x} = 0; x \to \infty, t \geq 0 \tag{5}$$

The physical system of the problem given in Figure 1.

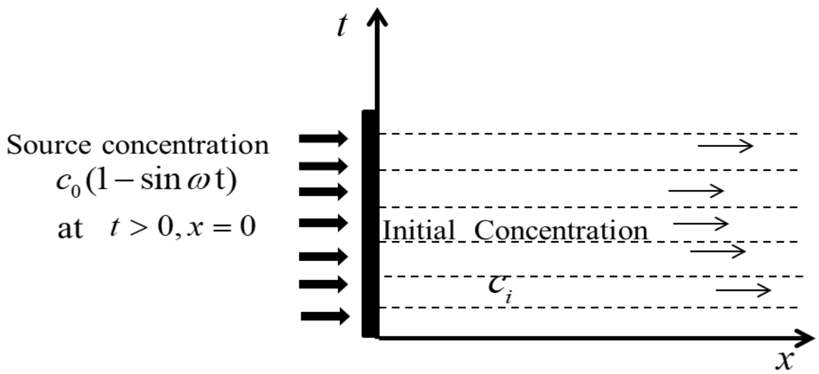

Figure 1. Physical system of the problem.

Using Equation (2), Equation (1) written as

$$R\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} - \frac{u}{\theta}\frac{\partial c}{\partial x} - \mu c \tag{6}$$

where, $R = \left(1 + \frac{\rho}{\theta}k_d\right)$ (7)

Retardation factor from which the rate of desorbed/sorbed solute on the solid liquid interphases is considered.

and $\mu = \left(\lambda k_d \frac{\rho}{\theta} - \frac{q}{\theta}\right)$ (8)

For exact solution here seepage velocity is straight related to the preliminary outflow velocity and dispersion coefficient is straight related to the initial outflow velocity (Freeze and Cherry, 1979),

$$u = u_0 f(mt) \tag{9}$$

and $D\alpha u$, i.e., $D = \alpha u$, where α is constant which depends upon the physical properties of the pore in homogeneous geological formation. After using the value of u, we have

$$D = D_0 f(mt) \tag{10}$$

and $\mu = \mu_0 f(mt)$ (11)

where, $u_0\left[LT^{-1}\right]$ and $D_0\left[L^2T^{-1}\right]$ are known as initial outflow velocity and initial dispersion measurement respectively, where, $m\left[T^{-1}\right]$ is the flow interrupt measurement.

Now, using Equations (9), (10) and (11), Equation (6) written as

$$\frac{R}{f(mt)}\frac{\partial c}{\partial t}=D_0\frac{\partial^2 c}{\partial x^2}-\frac{u_0}{\theta}\frac{\partial c}{\partial x}-\mu_0 c \tag{12}$$

By presenting a different time variable as

$$T^*=\int_0^t f(mt)dt \tag{13}$$

Equation (12) written as

$$R\frac{\partial c}{\partial T^*}=D_0\frac{\partial^2 c}{\partial x^2}-\frac{u_0}{\theta}\frac{\partial c}{\partial x}-\mu_0 c$$
(14)

By presenting non-dimensional variables as

$$C=\frac{c}{c_0},\ X=\frac{xu_0}{D_0},\ T=\frac{T^*u_0^2}{D_0},\ \mu^*=\frac{\mu_0 D_0}{u_0^2} \tag{15}$$

Equation (14) becomes,

$$R\frac{\partial C}{\partial T}=\frac{\partial^2 C}{\partial X^2}-\frac{1}{\theta}\frac{\partial C}{\partial X}-\mu^* C \tag{16}$$

The initial and boundary conditions given in Equation (3), Equation (4), and Equation (5) can be written in the non-dimensional form as:

$$C(X,T)=\frac{c_i}{c_0};\ X>0,\ T=0 \tag{17}$$

$$C(X,T)=\left[1-\sin\omega^* T\right];\ X=0,\ T\geq 0 \tag{18}$$

where, $\omega^*=\dfrac{\omega_0 D_0}{u_0^2}$.

$$\frac{\partial C}{\partial X}=0;\ X\to\infty, T\geq 0 \tag{19}$$

By presenting a different transformation as

$$C(X,T)=K(X,T)\exp\left[\frac{X}{2\theta}-\frac{1}{R\theta}\left(\frac{1}{4\theta}+\mu^*\theta\right)T\right] \tag{20}$$

Equation (16) becomes,

$$R\frac{\partial K}{\partial T}=\frac{\partial^2 K}{\partial X^2} \tag{21}$$

With the help of Laplace integral transform technique subjects to initial and boundary conditions given in

Equation (17) - (19) the desired closed form solution can be written as,

$$C(X,T)=\left\{A_1(X,T)-\omega^* A_2(X,T)-\frac{c_i}{c_0}A_3(X,T)+\frac{c_i}{c_0}\exp\left(\frac{1}{4R\theta^2}T-\frac{X}{2\theta}\right)\right\}\exp\left[\frac{X}{2\theta}-\frac{1}{R\theta}\left(\frac{1}{4\theta}+\mu^*\theta\right)T\right] \tag{22}$$

where,

$$A_1(X,T)=\frac{1}{2}\left[\exp\left(QT-\sqrt{QR}X\right)exfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}-\sqrt{QT}\right)\right]+\frac{1}{2}\left[\exp\left(QT+\sqrt{QR}X\right)exfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}+\sqrt{QT}\right)\right] \tag{23}$$

$$\begin{aligned}A_2(X,T)=&\frac{1}{4\sqrt{Q}}\left(2\sqrt{Q}T-\sqrt{R}X\right)\left[\exp\left(QT-\sqrt{QR}X\right)exfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}-\sqrt{QT}\right)\right]\\&+\frac{1}{4\sqrt{Q}}\left(2\sqrt{Q}T+\sqrt{R}X\right)\left[\exp\left(QT+\sqrt{QR}X\right)exfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}+\sqrt{QT}\right)\right]\end{aligned} \tag{24}$$

$$A_3(X,T)=\frac{1}{2}\left[\exp\left(\frac{T}{4R\theta^2}-\frac{X}{2\theta}\right)\right]erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}-\frac{\sqrt{T}}{2\sqrt{R}\theta}\right)+\frac{1}{2}\left[\exp\left(\frac{T}{4R\theta^2}+\frac{X}{2\theta}\right)\right]erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}+\frac{\sqrt{T}}{2\sqrt{R}\theta}\right) \tag{25}$$

$$Q=\frac{1}{R\theta}\left(\frac{1}{4\theta}+\mu^*\theta\right) \tag{26}$$

3. Results and Discussions

3.1 Interpretation of Results

The analysis of solute segmentation in groundwater movement depends upon the parameters of different geological formation such as porosity, density, decay constant term, first order decay term. The closed solution obtained in Equation (22) analysed for the given group of inputs data (Singh et al., 2009): $c_i = 0.01 mg/l$, $c_0 = 1.0 mg/l$, $u_0 = 0.1 km/\text{year}$, $D_0 = 0.8\ km^2/\text{year}$, $\rho = 1.80(slit), 1.63(clay), 1.65(gravel)$, $\theta = 0.35(slit),\ 0.40\ (clay),\ 0.32(gravel)$, $\lambda = 0.002/\text{year}$, $w^* = 0.007/\text{year}$, $q = 0.005/\text{year}$, $k_d = 0.5$, $k = 5.5$, $m = 0.03/\text{year}$.

The two different velocity pattern are used for graphical outcomes of results in Equation (22),

(i) Exponential decreasing pattern $f(mt)=e^{-kmt}$ (27)

$$T=\frac{{u_0}^2}{D_0 km}\left[1-e^{-kmt}\right] \tag{28}$$

(ii) Sinusoidal pattern $f(mt)=1-\sin(kmt)$ (29)

$$T=\frac{{u_0}^2}{D_0}\left[t+\frac{\cos kmt}{km}-\frac{1}{km}\right] \tag{30}$$

The analysis of solute segmentation distribution represented for fixed interval of domain $0 \le x \le 1$ at time interval $t = 1$ year, 2 year, 3 year respectively for distinct geological systems with their density and porosity values. The graphical representations of solute concentration C depicts against the distance X using MATLAB software.

3.2 Analysis & Discussions of Results

The solute segmentation distribution depicts for clay geological system for exponential decreasing velocity pattern with their averaging porosity and density values at time intervals t = 1 year, 2 year, 3 year respectively in Figure 2. Initially, at the geological formation the solute segmentation distribution originated from uniform source for all time periods. The solute segmentation distribution increases with increasing time values whereas the solute segmentation pattern follows declining character with respect to the space.

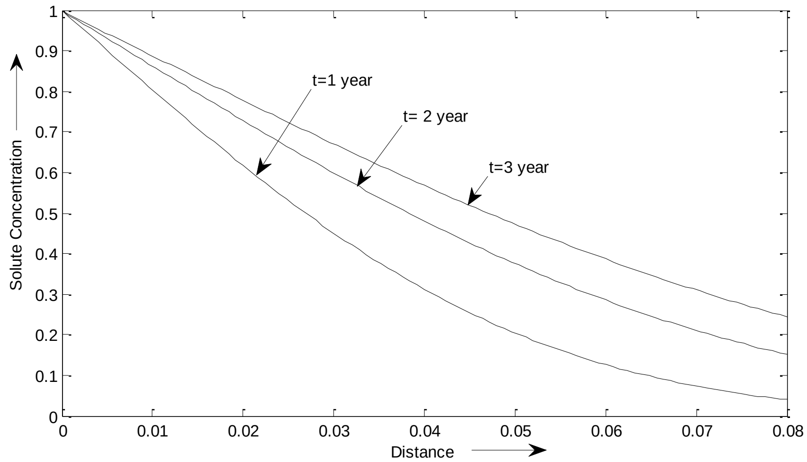

Figure 2. Solute segmentation pattern for clay geological formation for exponential decreasing velocity.

The solute segmentation distribution depicts for slit geological formation with their porosity and density values at time intervals t = 1 year, 2 year, 3 year respectively in Figure 3 for exponential decreasing velocity pattern. The solute segmentation distribution pattern increases with increasing time values whereas the solute segmentation pattern follows declining character with respect to the space. The declining character of solute segmentation faster for time t = 1 year as compare to the other solute segmentation pattern. However, at the end of geological formation solute segmentation values achieves its least segmentation values for all time periods.

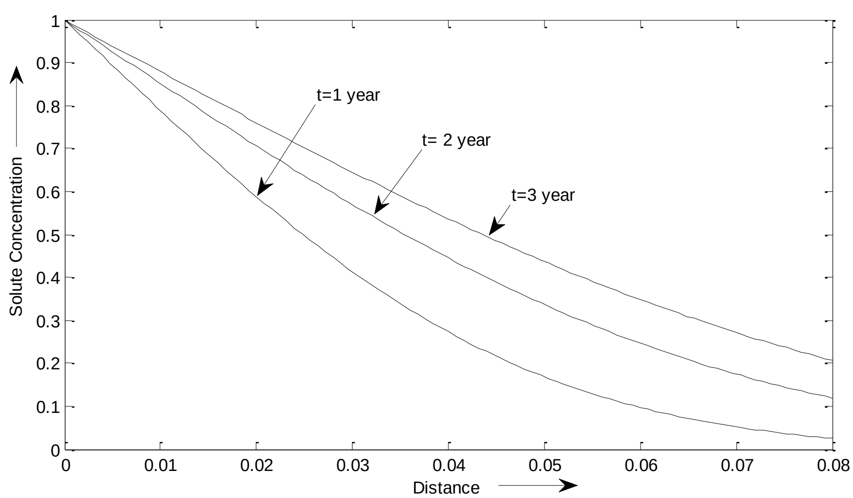

Figure 3. Solute segmentation pattern for slit geological formation for exponential decreasing velocity.

The solute segmentation distribution depicts for gravel geological formation with their porosity and density values at time interval $t = 1$ year, 2 year, 3 year respectively in Figure 4 for exponential decreasing velocity pattern. The solute segmentation distribution pattern increases with increasing time values whereas the solute segmentation pattern follows declining character with respect to the space. The decreasing nature of solute segmentation faster for time $t = 1$ year as compare to the other solute segmentation patterns. However, at the end of geological formation solute segmentation values achieves its least solute segmentation values for all time periods.

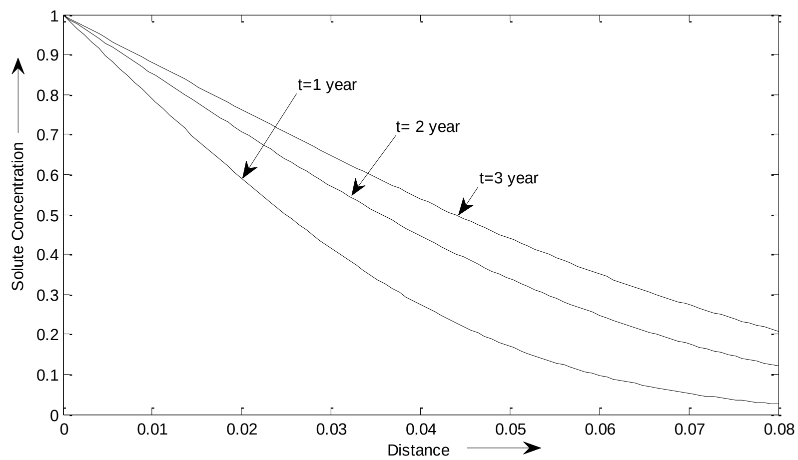

Figure 4. Solute segmentation pattern for gravel geological formation for exponential decreasing velocity.

The solute segmentation distribution depicts for gravel geological formation with their porosity and density values at time interval $t = 1$ year, 2 year, 3 year respectively in Figure 5 for sinusoidal velocity pattern. The solute segmentation distribution pattern increases with increasing time values whereas the solute segmentation pattern follows declining character with respect to the space. The decreasing nature of solute segmentation faster for time $t = 1$ year as compare to the other solute segmentation pattern. However, as compare the Figure 4 and Figure 5 the solute segmentation values for gravel medium attains the least values at each of the time periods in exponential decreasing velocity pattern as compare to sinusoidal velocity pattern.

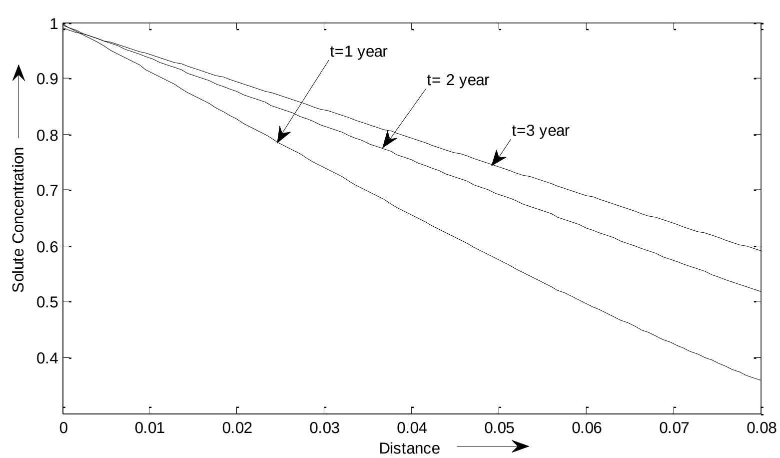

Figure 5. Solute segmentation pattern for gravel geological formation for sinusoidal velocity.

The velocity pattern of groundwater flow is an important role for solute segmentation distribution into groundwater reservoir. The presence of solute is depending upon the velocity of groundwater contaminant flow. The solute segmentation for distinct patterns of velocity of clay geological formation is depicts in Figure 6. The solute segmentation slightly takes minimum values at exponential decreasing velocity pattern for each of the time interval as compare to the sinusoidal velocity pattern. The solute segmentation distribution values take the minimum values at each point of space at the end of the geological formation for both the velocity patterns.

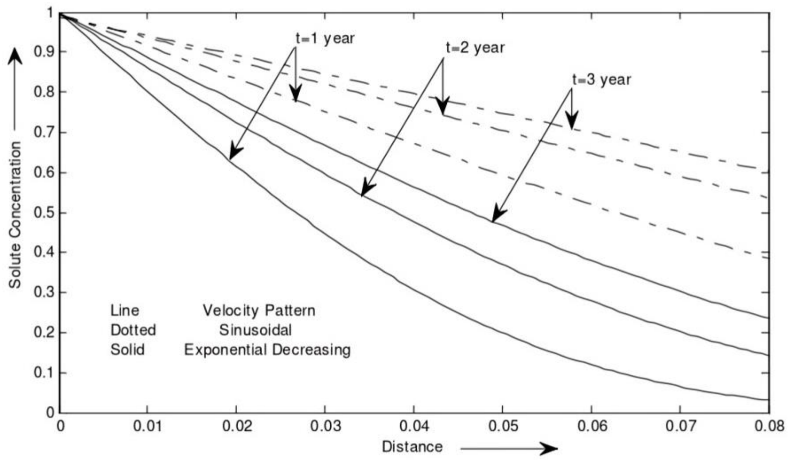

Figure 6. Comparison of contaminant segmentation distribution for clay geological formation with sinusoidal and exponential decreasing velocity pattern.

The average porosity and density values of distinct geological formations gives the impact of solute segmentation distribution pattern. The same minimum solute segmentation distribution attains in slit and gravel formations as compare to the clay formations. However, maximum solute segmentation patterns achieve in clay medium. However, from Figure 7 all the solute segmentation distribution pattern for clay, slit and gravel geological formation achieves its least harmless solute segmentation values at the end of the domain.

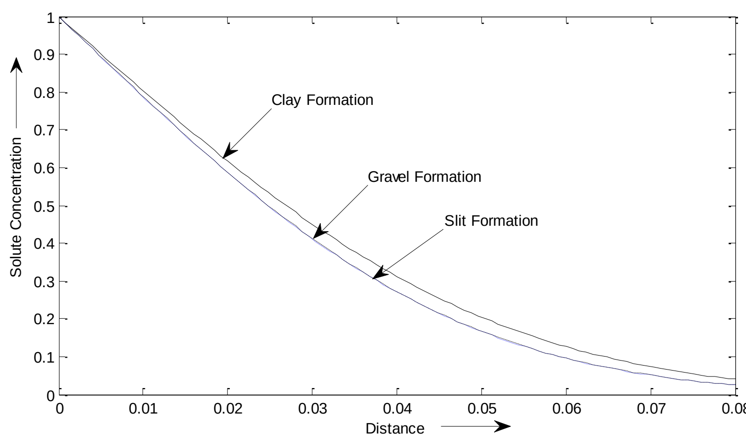

Figure 7. Comparison of contaminant segmentation pattern for different geological formation with uniform time.

The biodegradation decay parameter ω represents the fluctuation rate of biodegradation term presence in the geological formation. The solute segmentation predicts in Figure 8 with the varying value of biodegradation decay parameter for gravel geological formation for fixed time period $t = 1$ year. The solute segmentation variations follow same rate for both values of biodegradation decay parameter. Initially slight variation of solute concentration observed but beyond some distance it shows the same decreasing nature at each of the distance.

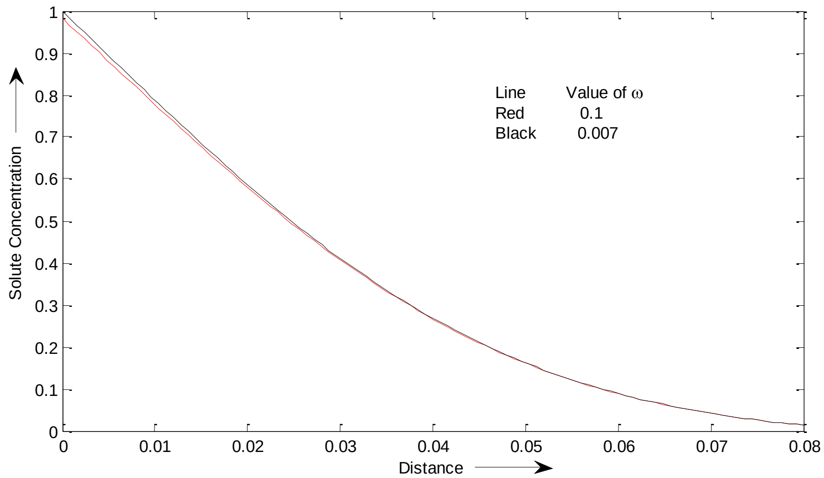

Figure 8. Comparison of contaminant segmentation pattern for different value of the zero order production rate.

The surface concentration distribution of the gravel medium is depicting in Figure 9 for exponential decreasing velocity pattern. It's observed that the solute segmentation increase in respect of time whereas decreasing trends shows in respect of distance.

Similarly, the surface concentration distribution of the clay medium is depicting in Figure 10 for exponential decreasing velocity pattern. It's observed that the solute segmentation increase in respect of time whereas decreasing trends shows in respect of distance.

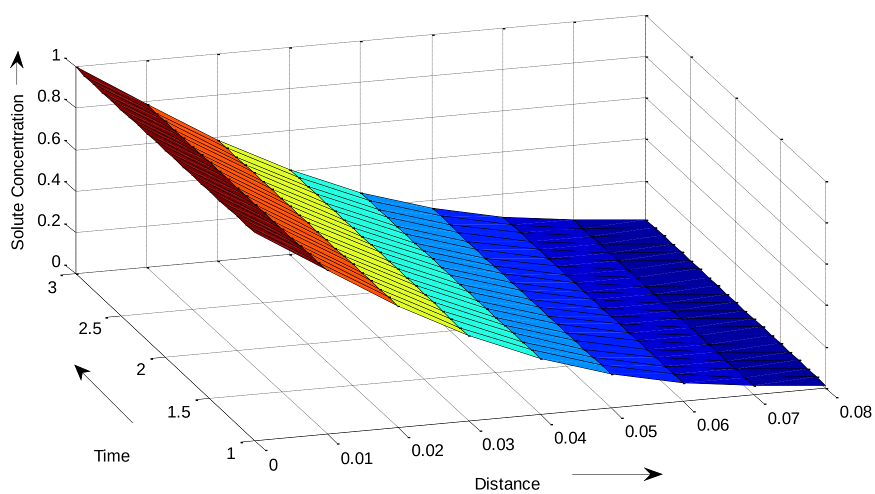

Figure 9. Surface concentration distribution pattern for gravel medium.

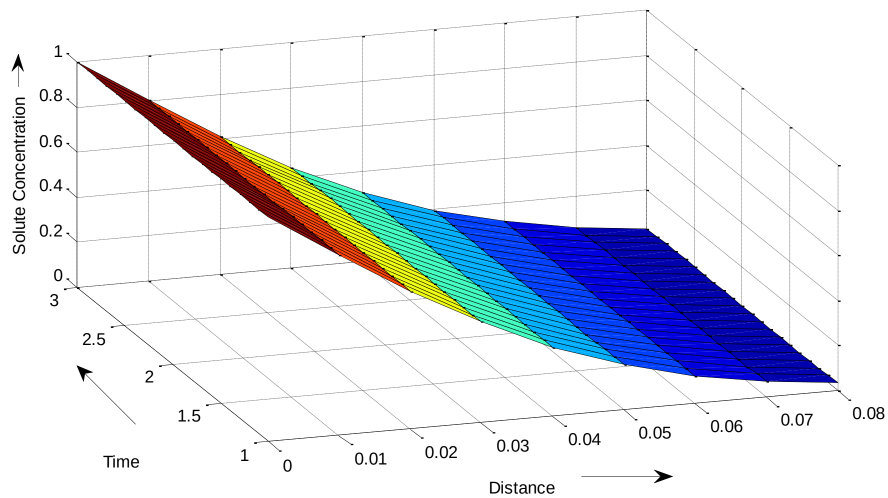

Figure 10. Surface concentration distribution pattern for clay medium.

The comparison of concentration values for the distinct geological formations (i.e.; gravel, slit and clay) are tabulated in respect of distinct velocity patterns in Table 1, Table 2 and Table 3 respectively. From Table 1 it is observed that the solute concentration values attain slightly higher concentration for sinusoidal velocity pattern as compare to the exponential decreasing velocity patter for each of the time period in respect of space. Similar nature of the concentration patterns is observed for the slit and clay formations where the concentration values attain slightly higher values in sinusoidal velocity pattern as compare to the exponential decreasing velocity pattern for each of the time period in respect of space.

Table 1. The comparison of concentration value for the gravel geological formation in respect of distinct velocity pattern.

Distance	Concentration					
	Time t = 1 year		Time t = 2 year		Time t = 3 year	
	Exponential decreasing velocity pattern	Sinusoidal velocity pattern	Exponential decreasing velocity pattern	Sinusoidal velocity pattern	Exponential decreasing velocity pattern	Sinusoidal velocity pattern
0.0008	0.9826	0.9909	0.9872	0.9898	0.9887	0.9862
0.02	0.5686	0.8239	0.7055	0.8736	0.76	0.8912
0.04	0.2686	0.6505	0.4414	0.7489	0.5339	0.7881
0.06	0.0892	0.4889	0.2399	0.6254	0.3424	0.6837
0.08	0.0138	0.3477	0.1087	0.5077	0.1972	0.5811

Table 2. The comparison of concentration value for the slit geological formation in respect of distinct velocity pattern.

Distance	Concentration					
	Time t = 1 year		Time t = 2 year		Time t = 3 year	
	Exponential decreasing velocity pattern	Sinusoidal velocity pattern	Exponential decreasing velocity pattern	Sinusoidal velocity pattern	Exponential decreasing velocity pattern	Sinusoidal velocity pattern
0.0008	0.9824	0.99	0.9869	0.9884	0.9882	0.9846
0.02	0.5858	0.8216	0.704	0.8706	0.7582	0.888
0.04	0.2681	0.6475	0.4399	0.7448	0.5318	0.7836
0.06	0.0896	0.486	0.2392	0.6209	0.3409	0.6785
0.08	0.0149	0.3454	0.109	0.5033	0.1966	0.5757

Table 3. The comparison of concentration value for the clay geological formation in respect of distinct velocity pattern.

Distance	Concentration					
	Time t = 1 year		Time t = 2 year		Time t = 3 year	
	Exponential decreasing velocity pattern	Sinusoidal velocity pattern	Exponential decreasing velocity pattern	Sinusoidal velocity pattern	Exponential decreasing velocity pattern	Sinusoidal velocity pattern
0.0008	0.9835	0.9896	0.9874	0.9871	0.9884	0.9826
0.02	0.615	0.8337	0.7253	0.8779	0.7754	0.8931
0.04	0.3078	0.672	0.4763	0.7616	0.5637	0.7966
0.06	0.1198	0.52	0.2788	0.6464	0.3802	0.6993
0.08	0.0309	0.3848	0.1422	0.536	0.2355	0.6036

3.3 Relative Percent Error

For testing the accuracy of solutions in this paper, we used the Relative Percent Error (RPE) which is significantly check the accuracy of the solution in respect of time period 1 year. The relative error was used to calculate the accuracy of the concentration segmentation in respect of the time period t= 1 year which is defined by,

$$RPE = \frac{|\exp ected\ value - True\ value|}{True\ value} \times 100 \tag{31}$$

The relative percent error is tabulated for each of the geological formations in respect of distinct velocity pattern. The relative error for the gravel medium attain higher value in case of exponential decreasing velocity pattern as compare to the sinusoidal velocity pattern for time interval t = 2 year and 3 year observed from Table 4. Similar nature of RPE obtained from Table 5 and Table 6 for the case of slit and clay medium for the time interval t = 2 year and 3 year respectively. The RPE value increases as increase the time period for respective velocity pattern in three distinct geological formations.

Table 4. Relative percent error for the gravel medium for exponential decreasing and sinusoidal velocity pattern.

Distance	Concentration values for exponential decreasing velocity pattern			Concentration values for sinusoidal velocity pattern		
	Time t=1 year	Time t=2 year	Time t=3 year	Time t=1 year	Time t=2 year	Time t=3 year
0.0008	0.9826	0.9872	0.9887	0.9909	0.9898	0.9862
0.02	0.5686	0.7055	0.76	0.8239	0.8736	0.8912
0.04	0.2686	0.4414	0.5339	0.6505	0.7489	0.7881
0.06	0.0892	0.2399	0.3424	0.4889	0.6254	0.6837
0.08	0.0138	0.1087	0.1972	0.3477	0.5077	0.5811
Relative Percent Error		29.11	46.77	Relative Percent Error	13.49	19.31

Table 5. Relative percent error for the slit medium for exponential decreasing and sinusoidal velocity pattern.

Distance	Concentration values for exponential decreasing velocity pattern			Concentration values for sinusoidal velocity pattern		
	Time t=1 year	Time t=2 year	Time t=3 year	Time t=1 year	Time t=2 year	Time t=3 year
0.0008	0.9824	0.9869	0.9882	0.99	0.9884	0.9846
0.02	0.5858	0.704	0.7582	0.8216	0.8706	0.888
0.04	0.2681	0.4399	0.5318	0.6475	0.7448	0.7836
0.06	0.0896	0.2392	0.3409	0.486	0.6209	0.6785
0.08	0.0149	0.109	0.1966	0.3454	0.5033	0.5757
Relative Percent Error		27.73	45.07	Relative Percent Error	13.39	19.16

Table 6. Relative percent error for the clay medium for exponential decreasing and sinusoidal velocity pattern.

Distance	Concentration values for exponential decreasing velocity pattern			Concentration values for sinusoidal velocity pattern		
	Time t=1 year	Time t=2 year	Time t=3 year	Time t=1 year	Time t=2 year	Time t=3 year
0.0008	0.9835	0.9874	0.9884	0.9896	0.9871	0.9826
0.02	0.615	0.7253	0.7754	0.8337	0.8779	0.8931
0.04	0.3078	0.4763	0.5637	0.672	0.7616	0.7966
0.06	0.1198	0.2788	0.3802	0.52	0.6464	0.6993
0.08	0.0309	0.1422	0.2355	0.3848	0.536	0.6036
Relative Percent Error		26.88	43.08	Relative Percent Error	12.17	17.32

3.4 Validation of Model

The validation of the developed model equation is shown with the existing research work carried out by Gharehbaghi (2016). Gharehbaghi (2016) explored the solution of one-dimensional solute transport model equation in semi-infinite porous media by using differential quadrature method. The Table 7 tabulated the different inputs values (authors input and Gharehbaghi, 2016) for the validation purpose. The concentration distribution pattern for different inputs values is predicted in Figure 11 for the exponential decreasing velocity pattern for clay geological formation. The concentration values for the input value (i) attain the minimum level of concentration as compared to the input (ii). As the solution developed in non-dimensional form so distance covered different for the different inputs values. Beyond some distance both the concentration values attain its minimum harmless concentration at the end of the geological formation.

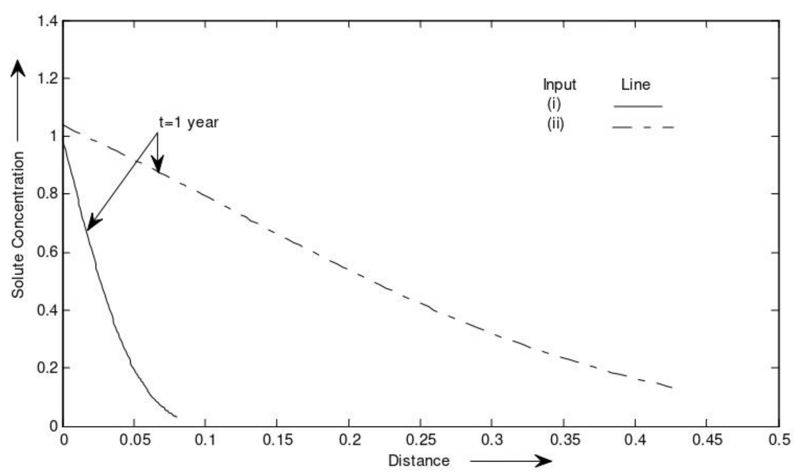

Figure 11. concentration segmentation pattern of clay medium for distinct set of input data.

Table 7. The different input values for validation purpose.

Author's Name	Input values		
	Source concentration (c_0)	Initial dispersion coefficient (D_0)	Initial seepage velocity (u_0)
(i)Author's Input	1.0	0.80	0.10
(ii) Gharehbaghi (2016)	1.0	0.71	0.60

4. Summary and Conclusions

The exact solution of one-dimensional solute transport model equation with the effect of the various decay and biodegradation decay parameters in homogeneous semi-infinite aquifer were found. The impact of distinct groundwater reservoirs properties such as porosities, densities and various decay order terms were taken into interpretation over the obtained solution.

(i) The impact of linear isotherm was analyzed for solute transport through solid liquid interphase. Also, solute dispersion coefficient was straight related to the initial outflow velocity incorporate so that contaminant level into groundwater reservoirs can be measure in respect of groundwater flow.

(ii) The nature of distinct geological formations was studied carefully. The solute contaminant level was observed minimum in slit and gravel medium as compared to the clay medium for both the exponential and sinusoidal velocity patterns.

(iii) The solute contaminant level was observed minimum level for each of the geological formation in exponential decreasing velocity as compared to the sinusoidal velocity pattern.

(iv) The surface concentration study for the gravel and clay medium was presented regarding solute transport against surface of the groundwater reservoir, and the variation of pollutant was shown in the groundwater reservoir surface.

(v) The accuracy rate in sinusoidal velocity pattern observed minimum as compare to the exponential decreasing velocity pattern. Also, the model validated with the existing research work and found that the concentration segmentation attains the same pattern, however the authors inputs covered the minimum harmless concentration at each of the point as compare to other one.

(vi) The proposed exact solution of the governing equation may be useful to predict the solute contaminant level in respect of time and distance in presence of decay parameter into the groundwater reservoir. It will become recyclable and useful for remediation of contaminant in urban and industrialized areas.

(vii) The governing equation with the surface source contamination may further be investigated for the solute segmentation nature in groundwater reservoir.

List of Symbols

D: Longitudinal dispersion coefficient; $\left[L^2T^{-1}\right]$.

u: Unsteady uniform pore seepage velocity; $\left[LT^{-1}\right]$.

c: The volume averaged dispersing solute concentration in the liquid phase; $\left[ML^{-3}\right]$.

s: The volume averaged dispersing solute concentration in the solid phase; $\left[ML^{-3}\right]$.

c_i: Initial concentration; $\left[ML^{-3}\right]$.

c_0: Source concentration; $\left[ML^{-3}\right]$.

λ: The decay rate constant; $\left[T^{-1}\right]$.

K_d: Distribution coefficient.

ρ: Bulk density; $\left[ML^{-3}\right]$

θ: Porosity of the different geological formation

α: Dispersivity.

D_0: Initial dispersion coefficient; $\left[L^2T^{-1}\right]$.

u_0: Initial seepage velocity; $\left[LT^{-1}\right]$.

n: Porosity of the different geological formation.

x: The longitudinal direction of flow; $[L]$.
m: The flow resistance coefficient; $\left[T^{-1}\right]$.
t : Time variable; $[T]$.
S: Laplace transforms parameter.
$\bar{K}$: Laplace transform of K .
C: Non dimensional solute concentration in geological formations.
X : Non dimensional direction of flow.
q : First order decay rate coefficients $\left[T^{-1}\right]$.
ω: Biodegradation decay rate coefficients; $\left[T^{-1}\right]$.
$f(mt)$: The generalised case of the time dependent function.
R: Retardation factor.

Appendix

After using the transformation (20) in Equation (26) together with the initial and boundary conditions (17) to (19), we obtain as follows:

$$R\frac{\partial K}{\partial T}=\frac{\partial^2 K}{\partial X^2} \tag{A1}$$

$$K(X,0)=\frac{c_i}{c_0}\exp\left(\frac{-X}{2\theta}\right); \qquad T=0 \;\; X>0 \tag{A2}$$

$$K(X,0)=\left(1-\sin\omega^* T\right)\exp\left[\frac{1}{R\theta}\left(\frac{1}{4\theta}+\mu^*\theta\right)T\right]; \;\; X=0,\; T>0 \tag{A3}$$

$$\frac{\partial K}{\partial X}=-\frac{K}{2\theta}; \qquad X\to\infty \;\; T\geq 0 \tag{A4}$$

Taking Laplace integral transform technique over Equation (A1) together with the initial condition we obtain the complete solution as

$$\bar{K}(X,S)=Ae^{\sqrt{SR}X}+Be^{-\sqrt{SR}X}+\frac{c_i}{c_0}\frac{1}{\left(S-\frac{1}{4R\theta^2}\right)}\exp\left(\frac{-X}{2\theta}\right) \tag{A5}$$

where, $\overline{K}(X,S)=\int_0^\infty K(X,T)e^{-ST}dT$; A and B are the arbitrary constants.

From Equation (A3) and (A4), we get,

$$A=0 \tag{A6}$$

$$B=\frac{1}{(S-Q)}-\omega^*\frac{1}{(S-Q)^2}-\frac{c_i}{c_0}\frac{1}{\left(S-\frac{1}{4R\theta^2}\right)} \tag{A7}$$

where, $Q=\frac{1}{R\theta}\left(\frac{1}{4\theta}+\mu^*\theta\right)$.

Now using the values of A and B in Equation (A5), we obtain,

$$\bar{K}(X,S)=\frac{1}{(S-Q)}e^{-\sqrt{SR}X}-\omega^{*}\frac{1}{(S-Q)^{2}}e^{-\sqrt{SR}X}-\frac{c_i}{c_0}\frac{1}{\left(S-\frac{1}{4R\theta^{2}}\right)}e^{-\sqrt{SR}X}+\frac{c_i}{c_0}\frac{1}{\left(S-\frac{1}{4R\theta^{2}}\right)}\exp\left[\frac{-X}{2\theta}\right] \tag{A8}$$

Now taking the inverse Laplace transform of each terms of Equation (A8) we get the desired result

$$K(X,T)=\left\{A_1(X,T)-\omega^{*}A_2(X,T)-\frac{c_i}{c_0}A_3(X,T)+\frac{c_i}{c_0}\exp\left(\frac{1}{4R\theta^{2}}T-\frac{X}{2\theta}\right)\right\} \tag{A9}$$

Using Equation (A9) in Equation (20) we get the desired result which we find from Equations (22) and (24).

Conflict of Interest

The authors declare that there is no conflict of interest.

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